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1. Introduction

Since 2013, the value of mineral production in the country has been falling due to the contraction of the price of ore in the global market, and as a result, good initial planning becomes increasingly crucial to the life of an open-pit mine when treated within the scope of its economic viability. In this sense, the present study aims to present a model for the problem of open-pit mine scheduling, considering its dynamic and stochastic aspects, in order to generate a framework that will serve as the basis for the evaluation, through simulation, of different solution policies. The proposed solution considers the uncertainties of the problem regarding the mineral price and geological uncertainty aspects and, in order to demonstrate the application of the proposed method, a demonstrative problem is presented and solved through a myopic solution policy.

2. Mathematical Modeling and Description

For mathematical modeling purposes, we will represent an open pit as a set of $\mathcal{B}$ ore blocks arranged in $\mathbb{R}^3$. Each $i \in \mathcal{B}$ element has associated a set $\mathcal{B}_i \subseteq \mathcal{B}$ of blocks that need to be extracted before i so that the latter becomes available. Thus, $\mathcal{B}_i$ represents the set of precedents of $i \in \mathcal{B}$. In order to decompose the problem, the blocks are aggregated into partitions $r \in \mathcal{R}$, that is, $\bigcup_{r \in \mathcal{R}} \mathcal{B}_r = \mathcal{B}$, with $\mathcal{B}_{r_1} \cap \mathcal{B}_{r_2} = \emptyset,\ \forall (r_1, r_2) \in \mathcal{R}$ and $r_1 \neq r_2$. The size reduction through block aggregation aims to limit the load of the problem to the available blocks for extraction, called $\mathcal{R}_d$, concept that will be defined later.

Each block has a cost of extraction c_i and a sterile/ore ratio e_i unknown until the time of extraction. The combination of c_i, e_i and the price of ore in market p, determines the value of block v_i at the time of extraction. During the planning stage, the values v_i and e_i are estimated by $\hat{v}_i$ and $\hat{e}_i$, respectively.

The objective of the problem is to sequence the extraction of the blocks $i \in \mathcal{B}$ in order to maximize the net present value of the activity. For this purpose, capacity constraints, minimum extracted value and extraction precedence are considered for each decision period, that is, for each moment in which one has the possibility to redo the extraction plan.

2.1 System state and decision stage

In the present work, the state of the system in stage n will be represented as $S^{(n)}$ and will contain the following parameters:

- Available aggregate blocks: $\mathcal{R}_d^{(n)}$ represents the aggregate blocks available for exploration in the decision stage n;
- Blocks with possibility of exploration: the sets $\mathcal{B}^{(n)}$ and $\mathcal{B}_r^{(n)}, \forall r \in \mathcal{R}_d^{(n)}$ are the blocks that can be mined in the decision stage n;
- Estimation of the sterile/ore ratio: the parameter $\hat{e}_i^{(n)}$ reports the sterile/ore estimate of the block i in the n stage of decision;
- Value of the ore in the market: the parameter $p^{(n)}$ refers to the value of the ore in the decision stage n;
- Benefit obtained with the extraction of the blocks: the parameter $\hat{v}_i^{(n)}$ expresses the value of the extraction of the block i in the decision stage n, given its relation of the sterile/ore $\hat{e}_i^{(n)}$ and the price $p^{(n)}$ of the current ore in the market.

Thus, the state of the system in stage n is represented by the Expression 1:

$$S^{(n)} = (\mathcal{R}_d^{(n)}, \mathcal{B}^{(n)} = \bigcup_{r \in \mathcal{R}_d^{(n)}} \mathcal{B}_r^{(n)}, v_i^{(n)}, e_i^{(n)}, p^{(n)}) \tag{1}$$

2.2 Uncertainties

The return generated with the extraction of geological blocks in an open-pit mine process has uncertain characteristics since its value is dependent on information regarding the price of the ore in stage n and the ratio of sterile/ore of the block, which is known with precision only at the moment of extraction. This information is considered exogenous to the ore extraction process.

$$w^{(n+1)} = (\bar{p}^{(n+1)}, \bar{e}_i^{(n)} \ \ \forall i \in \mathcal{B}_z^{(n)}) \tag{2}$$

2.3 Decision

For each block $i \in \mathcal{B}^{(n)}$ possible to be extracted at the current decision stage, a binary variable $x_i^{(n)}$ is associated indicating that this block was mined in n. Once extracted, the block immediately generates a value $\bar{v}_i$, now known, referring to the gain from its mining.

Being that $u^{(n)}$ is a feasible decision for the $S^{(n)}$ state in the stage n, the return at the end of the stage coming from the application of $u^{(n)}$ is given by Expression 3.

$$G_n(S^{(n)}, u^{(n)}) = \sum_{i \in \mathcal{B}_d^{(n)}} v_i * x_i \tag{3}$$

Since $U^{(n)}$ is the feasibility set for the decisions in the stage n, $\Omega = \{\omega^1, ..., \omega^{|K|}\}$ is the uncertainty path and λ a discount factor used to penalize future earnings, the objective of the problem modeled in this study can be represented by the Expression 4.

$$Obj: \max_{u^{(n)} \in U^{(n)}} \left\{ \mathbb{E}_\Omega \left[\sum_{n \in K} \lambda^n G_n(S^{(n)}, u^{(n)}) \right] \right\} \tag{4}$$

At each stage, the feasibility region of solutions $U^{(n)}$ is defined by the set of inequalities (5 - 11).

$$x_i^{(n)} \leq y_r^{(n)}, \forall r \in \mathcal{R}_d^{(n)}, \forall i \in \mathcal{B}_r^{(n)} \tag{5}$$

The constraint 5 informs that for a block to be extracted, the exploration of its aggregate block must be enabled. The binary variable $y_r^{(n)}, \forall r \in \mathcal{R}_d^{(n)}$ indicates the activation of the aggregate block r.

$$x_i^{(n)} \leq x_j^{(n)}, \forall i \in \mathcal{B}_r^{(n)}, \forall j \in \mathcal{P}_i^{(n)} \tag{6}$$

$$\sum_{i \in B_d^{(n)}} a_{ik} x_i^{(n)} \leq C_k^+, \forall \boldsymbol{k} \in \mathcal{K} \tag{7}$$

$$\sum_{i \in B_d^{(n)}} a_{ik} x_i^{(n)} \geq C_k^-, \forall \boldsymbol{k} \in \mathcal{K} \tag{8}$$

The constraint 6 ensures that the precedence relation between blocks is preserved. Since a_{ik} is the resource requirement of $k \in \mathcal{K}$ for the mining of the i block, the inequalities 7 and 8 the upper and lower limits of the use of each type of resource.

In order to reduce the risks of generating periods with negative gains, there should be constraints controlling the aspects related to tapering the mine at the time of extraction. The constraints 9 and 10 guarantee the precedence relation between the blocks aggregated to the point that an aggregate block must be exploited in its entirety before a next aggregate block is exploited and additionally minimize the incidence as the blocks are aggregated considering the coverage of negative extraction values (see Section 3.1), being that M_r represents the binary decision variable of which constraint should be used.

$$\sum_{r \in \mathcal{R}_d^{(n)}} y_r^{(n)} \leq 1 + \sum_{r \in \mathcal{R}_d^{(n)}} M_r \tag{9}$$

$$M_r \leq \frac{\sum_{i \in \mathcal{B}_r^{(n)}} x_i^{(n)}}{\mid \mathcal{B}_r^{(n)} \mid}, \forall r \in \mathcal{R}_d^{(n)} \tag{10}$$

Finally, the equations in 11 inform the domains of the variables.

$$x_i^{(n)} \in \{0,1\} \ \forall i \in \mathcal{B}_r^{(n)}, \ y_r^{(n)} \in \{0,1\} \ \ \forall r \in \mathcal{R}_d^{(n)}, \ \ M_r \in \{0,1\} \ \ \forall r \in \mathcal{R}_d^{(n)} \tag{11}$$

Since the decision is defined by a policy $\pi(\cdot)$, i.e., $u^{(n)} = \pi(S^{(n)})$, where $S^{(n)}$ is a possible state to occur under $\pi()$, Ω is the set of all possible uncertainty sequences for the system and $S^{(n+1)}$ is a state is possible to occur from the transition generated by S, u and ω, the Expression 12 represents the Bellman Equation for the π policy and derives the value $V^{\pi,(n)}$ to be in $S^{(n)}$.

$$V^{\pi,(n)}(S^{(n)}) = G(S^{(n)}, \pi(S^{(n)})) + \lambda E_\omega \{ V^{\pi,(n+1)}(S^{(n+1)}) | S^{(n)}, \pi(S^{(n)}) \} \tag{12}$$

In this way, the objective of the problem is to find the optimal policy $\pi^* = (\pi^1, ..., \pi^{(|K|)})$ from a set of policies Π that maximizes the value of being in the initial state $S^{(0)}$ in stage 0, according to the Expression 13.

$$\pi^* = \arg\max_{\pi \in \Pi} V^{\pi,(0)}(S^{(0)}) \tag{13}$$

2.4 Transition

The transition function $f_T(S, u, w)$ determines how the system evolves along the decision stages and, thus, it determines the dynamic of the process. Since $S^{(n)}$ is the present state, its configuration depends on the state of the system in the previous stage $S^{(n-1)}$, of the decision $u^{(n-1)}$ in the previous stage and also the influence of the uncertainty $w^{(n)}$ occurred in the current stage.

In this way, the dynamics described by Equation 14 represents the evolution of the system after the application of the control $u^{(n)}$ in stage n to a next state, existing in the stage $n+1$. In the Expression 14, $f(S^{(n)}, u^{(n)}, w^{(n+1)})$ represents a recursive function for the state in period $n+1$, based on past information.

$$S^{(t+1)} = f(S^{(t)}, u^{(t)}, \omega^{(t+1)}) \tag{14}$$

3. Application Example

In order to provide a better understanding of the proposed model, this section presents an application example that aims to demonstrate the dynamics of the decision throughout the stages.

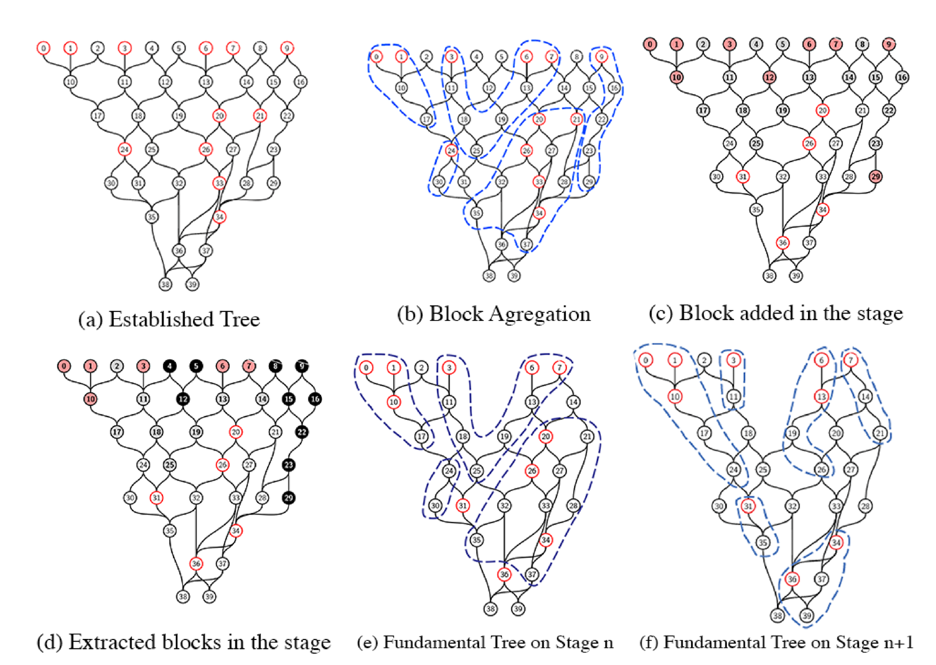
3.1 Aggregation of blocks

This section seeks to present the way in which the blocks are aggregated, in order to reduce the size of the problem.

In this paper, a simplification of the algorithm proposed by Ramazan [2007] is used. The author proposed a technique of resolution called Fundamental Tree Algorithm that performs the aggregation of blocks into larger blocks, dividing the problem tree into several smaller trees, in which the aggregation is carried out as a linear programming problem.

3.2 Production Scheduling

This sections demonstrates the decision-making process concerning the actions regarding the selection of the aggregate blocks to be explored in the current stage using a myopic policy and, after this event, the selection of the blocks to be extracted and the transition dynamics for the next period.



(a) Established Tree
(b) Block Agregation
(c) Block added in the stage
(d) Extracted blocks in the stage
(e) Fundamental Tree on Stage n
(f) Fundamental Tree on Stage n+1

Figure 1: *Application Example*

4. Results and discussions

In this study, a mathematical model for the resolution of the open-pit mine scheduling problem was presented. The contribution of this model is justified by the consideration of the stochastic and dynamic aspects of the problem acknowledging the value of the ore and the concentrations of the estimated sterile/ore ratio at the initial moment of extraction. A secondary contribution of the present study consists of the proposed formulation considering aggregation block evaluation methods, which seek to enable large-scale applications in severe cases. Still, a framework to evaluate via simulation different policies was constructed, making possible the evaluation of different exploration policies.

For future studies, it is made necessary the development of solution techniques for the proposed model and, posteriorly, your evaluation with other already applied techniques, as (e.g. Jélvez et al. [2016]; Ramazan [2007]). Together, it is suggested the experimentation of modeling referring to the uncertainties of the problem with methods present in literature (e.g. Dimitrakopoulos et al. [2002]; Chatterjee et al. [2016]). Finally, improvements for the aggregation model become relevant for better behavior of the proposed model.

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